

Predicate Logic: Quantifiers

- syntax
- base cases in programming

$$\forall i \bullet R(i) \Rightarrow P(i)$$

$$\exists i \bullet R(i) \wedge P(i)$$

Logical Quantifiers: Examples

$$\forall i \bullet i \in \mathbb{N} \Rightarrow i \geq 0$$

$$\forall i \bullet i \in \mathbb{Z} \Rightarrow i \geq 0$$

$$\forall i, j \bullet i \in \mathbb{Z} \wedge j \in \mathbb{Z} \Rightarrow i < j \vee i > j$$

$$\exists i \bullet i \in \mathbb{N} \wedge i \geq 0$$

$$\exists i \bullet i \in \mathbb{Z} \wedge i \geq 0$$

$$\exists i, j \bullet i \in \mathbb{Z} \wedge j \in \mathbb{Z} \wedge (i < j \vee i > j)$$

Logical Quantifiers: Examples

How to prove $\forall i \bullet R(i) \Rightarrow P(i)$?

How to prove $\exists i \bullet R(i) \wedge P(i)$?

How to disprove $\forall i \bullet R(i) \Rightarrow P(i)$?

How to disprove $\exists i \bullet R(i) \wedge P(i)$?

Prove/Disprove Logical Quantifications

• Prove or disprove: $\forall x \bullet (x \in \mathbb{Z} \wedge 1 \leq x \leq 10) \Rightarrow x > 0$.

• Prove or disprove: $\forall x \bullet (x \in \mathbb{Z} \wedge 1 \leq x \leq 10) \Rightarrow x > 1$.

• Prove or disprove: $\exists x \bullet (x \in \mathbb{Z} \wedge 1 \leq x \leq 10) \wedge x > 1$.

• Prove or disprove that $\exists x \bullet (x \in \mathbb{Z} \wedge 1 \leq x \leq 10) \wedge x > 10$?

Logical Quantifications: Conversions

$R(x): x \in 4315_class$

$P(x): x \text{ receives } A+$

$$(\forall x \bullet R(x) \Rightarrow P(x)) \Leftrightarrow \neg(\exists x \bullet R(x) \wedge \neg P(x))$$

$$(\exists x \bullet R(x) \wedge P(x)) \Leftrightarrow \neg(\forall x \bullet R(x) \Rightarrow \neg P(x))$$

De Morgan